ON THE PROBLEM OF THE DISINTEGRATION OF ROCK BY AN EXPLOSION UNDER THE INFLUENCE OF A FREE SURFACE

N. I. Shishkin

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1. When an explosion takes place underground, an intensive disintegration of the soil, known as spalling, takes place in the region of the epicenter of the explosion. Spalling also occurs in the case of an ejection explosion which precedes the stage of dispersal of the soil.

Spalling disintegration takes place because the compression-stress wave emitted by the focus of the explosion interacts with the free surface of the medium; in this interaction the compression wave is converted into a tension wave.

Soils and rocks have low stability with respect to tension in comparison to their stability with respect to compression. For this reason, soil not disintegrated in a compression wave which is already attenuated disintegrates easily in a tension wave of the same amplitude. It is the tension wave that causes the impressive scale of the spalling phenomenon in underground explosions. For example, it is shown in [1] that the volume of disintegration in the spalling zone may be twice as great as the volume of distintegration near the center of the explosion.

In order to study spalling, we shall consider the rock to be a linearly ideal elastic solid, and the concentrated explosion to be a point source which generates a spherically diverging elastic stress wave. This is the way the explosion problem was formulated in [2-4]. Onis'ko and Shemyakin [2] obtained and investigated formulas for the displacement band velocity of the free surface of an elastic half-space. Gutova and Nikiforovskii [3, 4] obtained and partially investigated formulas for the stress in a half-space.

In the present study, the problem of spalling in an underground explosion is solved for elastic solids whose stability with respect to tension is much less than their stability with respect to compression and shear. Such materials include, for example, soils and rocks (to the extent that they can be regarded as linearly elastic solids).

The elastic wave occurring when an explosion takes place in an unbounded medium will be described by the potential of the field of displacements $\varphi_0(R, t)$ in the form

$$\varphi_0(R, t) = \frac{A}{R} f\left[\frac{1}{t_0}(t - R/c_p)\right], \quad t \geqslant 0, \; R > 0, \tag{1.1}$$

where t is time; t_0 is the characteristic time; R is the distance from the center of the explosion; A is a constant which depends on the nature of the medium and the explosion; f is the form of the emitted wave; and c_p is the velocity of the longitudinal waves.

In [5] it was shown that a satisfactory approximation of experimental data on explosion waves can be obtained with the source function

$$f(x) = 1 - e^{-x}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} - Bx^4\right), \tag{1.2}$$

where B is a constant which depends on the properties of the medium.

The function f(x) satisfies the conditions

$$f(0) = f'(0) = f''(0) = f'''(0) = 0, \; f(\infty) = 1,$$

which signify the vanishing of the potential, the displacement, the velocity, and the acceleration on the wave front and the boundedness of f(x) at infinity.

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2. Now suppose that the source (1.1), (1.2) acts in a half-space which has a bounded free surface (the coordinate system is cylindrical, $r \geqslant 0,\ z \geqslant 0$, the free surface coincides with the plane z = 0, and the source is placed at the point r = 0, z = h).

The potentials of the displacement field in the half-space $\varphi(r, z, t)$ and $\psi(r, z, t)$ for this problem may be found in [6] (see also [2, 7, 8]):

$$\varphi(r, z, t) = \varphi_0(r, z_1, t) - \varphi_0(r, z_2, t) + \varphi_1(r, z_2, t), \tag{2.1}$$

$$\varphi(r, z, t) = -\frac{f(t-\rho)}{\rho}, \quad \rho = (r^2 + z^2)^{1/2}, \quad z_1 \equiv z - h, \ z_2 \equiv z + h,$$

$$\varphi_1(r, z_2, t) = \gamma \int_0^\infty kJ_0(kr) \left[\frac{1}{2\pi i} \int_l F(k\gamma\zeta) X(\zeta) e^{-kg_1(\zeta)} d\zeta \right] dk,$$

$$X(\zeta) = \frac{8\beta}{\delta^2 - 4\alpha\beta}, \ g_1(\zeta) = z_2\alpha - \gamma t\zeta;$$

$$\psi(r, z_2, t) = \gamma \int_0^\infty kJ_1(kr) \left[\frac{1}{2\pi i} \int_l F(k\gamma\zeta) Y(\zeta) e^{-kg_2(\zeta)} d\zeta \right] dk, \tag{2.2}$$

$$Y(\zeta) = \frac{4(2+\zeta^2)}{\delta^2 - 4\alpha\beta}, \quad g_2(\zeta) = h\alpha + z\beta - \gamma t\zeta,$$

$$\delta = 2 + \zeta^2, \ \alpha = \sqrt{1 + \gamma^2\zeta^2}, \ \beta = \sqrt{1 + \zeta^2}, \ \mathrm{Re}\,\alpha > 0, \ \mathrm{Re}\,\beta > 0, \ \text{if} \ \zeta > 0,$$

$\gamma = c_s/c_p$; c_s is the velocity of the transverse waves; $F(k\gamma\zeta)$ is the Laplace mapping of the source function f(t); J_0 and J_1 are Bessel functions; l is the countour of integration in the inversion formula for the Laplace transform [9]. Unlike the reference mentioned, in formula (2.1) we distinguish explicitly the term $\varphi_0(r, z_2, t)$, which corresponds to the potential of an imaginary source, as a result of which the term $\varphi_1(r, z_2, t)$ has no spatial singularities. In (2.1), (2.2), and everywhere in what follows, we introduce the dimensionless variables and quantities

$$t = \tilde{t}/t_0, \ r = \tilde{r}/c_p t_0, \ z = \tilde{z}/c_p t_0, \ h = \tilde{h}/c_p t_0, \ \varphi = \tilde{\varphi}/\varkappa(c_p t_0)^2, \ \varkappa = A/(c_p t_0)^3,$$

where the tilde indicates dimensionless quantities.

From (2.1) and (2.2) we can obtain formulas for the components of the stresses, as was done, for example, in [3, 4]. However, the formulas obtained are rather cumbersome, and their analysis requires extensive machine calculations. Evidently an exact elastic solution contains many precise details which are associated with the selected model of the medium and do not appear or have only a slight significance in the case of a real medium. It is desirable to simplify the solution by setting apart its principal part. If we confine our attention to the question of spalling disintegration of soils of the rock type, this can be done by starting from the characteristic strength properties of the rock and the criteria for its disintegration.

3. One of the characteristic properties of rocks is their fissibility. The cracks in the rock form a three-dimensional network separating the rock into blocks [10]. Consequently, the strength of rocks depends to a great extent on the nature of their stressed state: Rocks have the greatest strength under compression and the lowest strength under tension. For example, granites are characterized by approximately the following values of relative strength: 100 under compression, 9 under shear, and 2 under tension [11]. For media with different values of the ultimate strength under tension and under compression, the strength criterion has the form [12]

$$A(\sigma_{11} + \sigma_{22} + \sigma_{33}) + [B(\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2 + 2\sigma_{12}^2 + 2\sigma_{23}^2 + 2\sigma_{31}^2) + C(\sigma_{11} + \sigma_{22} + \sigma_{33})^2]^{1/2} \leqslant 1, \tag{3.1}$$

where

$$A = \frac{1}{2}\left(\frac{1}{\sigma^+} - \frac{1}{\sigma^-}\right); \quad B = \frac{1}{2\tau_b^2}; \quad C = \frac{1}{4}\left(\frac{1}{\sigma^+} + \frac{1}{\sigma^-}\right)^2 - \frac{1}{2\tau_b^2};$$

σ^+, σ^-, τ_b are the ultimate strengths under tension, compression, and shear, respectively.

The condition (3.1) (if the equal sign applies) may be interpreted as a disintegration criterion.

We introduce the notation: $m_1 = \sigma^+/\sigma^-$, $m_2 = \sigma^+/\tau_b$,

$$p = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}), \quad J_2 = \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2 - (\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11}).$$

In the new notation the disintegration criterion has the form

$$\frac{3}{2}pM = \sigma^+, \tag{3.2}$$

$$M \equiv 1 - m_1 + \left[(1+m_1)^2 - \frac{16}{9}m_2^2\right]^{1/2}\left[1 + \frac{4m_2^2}{(1+m_1)^2 - 16m_2^2/9}\frac{J_2}{9p^2}\right]^{1/2}.$$

For rocks, as mentioned earlier, $m_1 \leqslant 10^{-1}$, $m_2 \leqslant 2\cdot 10^{-1}$, i.e., m_1 and m_2 are much less than unity. Taking account of this fact, we can write the criterion (3.2) in the form

$$\frac{3}{2}p\left[2 + O\left(m_2^2\frac{J_2}{9p^2}\right)\right] = \sigma^+ \tag{3.3}$$

and take

$$p = (1/3)\sigma^+. \tag{3.4}$$

Thus, from the criterion (3.1) as applied to rocks it follows that a decisive role in their disintegration is played by omnidirectional tension.

The condition (3.4) is an idealized condition for the disintegration of a rock. It means that the rock is characterized by infinite strength under compression and shear and a finite strength σ^+ under simple tension.

It should be noted that for a plane longitudinal stress wave in an elastic medium (when the Poisson coefficient ν varies from 0 to $^1/_2$) the ratio $J_2/9p^2$ in (3.3) varies from 0 to $^1/_3$:

$$0 \leqslant \left|\frac{J_2}{9p^2}\right| = \frac{(2-\nu)\nu}{(1+\nu)^2} \leqslant \frac{1}{3}.$$

4. When we use the disintegration criterion (3.4) for a rock in the spalling problem, we must know the expression for the average stress as a function of the coordinates and time. For this, it is sufficient to know the longitudinal potential $\varphi(r, z, t)$ alone, since

$$p(r, z, t) = \frac{3-4\gamma^2}{3}\frac{\partial^2\varphi}{\partial t^2}. \tag{4.1}$$

Substituting into (4.1) the value of φ from (2.1), we obtain

$$p(r, z, t) = \frac{3-4\gamma^2}{3}\left\{-\frac{f''(t-\rho_1)}{\rho_1} + \frac{f''(t-\rho_2)}{\rho_2} + \gamma\int_0^\infty kJ_0(kr)\left[\frac{1}{2\pi i}\int_l X(\zeta)\,\gamma^2k^2\zeta^2F(k\gamma\zeta)\,e^{-kg_1(\zeta)}d\zeta\right]dk\right\}, \tag{4.2}$$

$$\rho_1 = (r^2 + z_1^2)^{1/2}, \quad \rho_2 = (r^2 + z_2^2)^{1/2}.$$

Deforming the contour l into a new contour which encloses the branching lines drawn along the imaginary axis from the points $\zeta = \pm i$ to infinity and taking account of the residues at the poles $\zeta = 0$, $\zeta = \pm i\vartheta$, we can obtain the following representation of p:

$$p = p_a + p_0 + p_1 + p_2, \quad p_a = \frac{3-4\gamma^2}{3}\left[-\frac{f''(t-\rho_1)}{\rho_1} + \frac{f''(t-\rho_2)}{\rho_2}\right], \tag{4.3}$$

$$p_0 = \frac{3-4\gamma^2}{3}\frac{4\gamma^2}{1-\gamma^2}\left(-1 + \frac{3z_2^2}{\rho_2^2}\right)\left[\frac{f'(t-\rho_2)}{\rho_2^2} + \frac{f(t-\rho_2)}{\rho_2^3}\right],$$

$$p_1 = \frac{3-4\gamma^2}{3}\Phi(\vartheta)\int_{\rho_2}^{t} S(r, z_2a, \gamma\vartheta\tau)\,f''(t-\tau)\,d\tau,$$

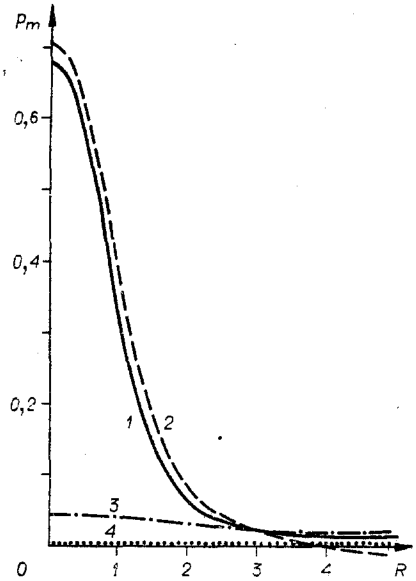

Fig. 1

$$p_2 = -\frac{3-4\gamma^2}{3}\frac{\gamma}{\pi}\int\limits_{\rho_2}^{t} f''(t-\tau)\int\limits_{1}^{1/\gamma} S(r,\, z_2\alpha,\, \gamma\tau\lambda)\, Q(\lambda)\, d\lambda d\tau,$$

$$\Phi(\vartheta) = \frac{4\gamma a b^2}{\vartheta\Theta(\vartheta)}, \quad \Theta(\vartheta) = abd - (a^2+\gamma^2 b^2),$$

$$a = \sqrt{1-\gamma^2\vartheta^2}, \quad b = \sqrt{1-\vartheta^2}, \quad d = 2-\vartheta^2,$$

$$S(r,\, p,\, q) \equiv \frac{q(3AB^2 - A^3) + p(3BA^2 - B^3)}{R^3},$$

$$A = \left(\frac{R+X}{2}\right)^{1/2}, \quad B = \left(\frac{R-X}{2}\right)^{1/2}, \quad R = (X^2+Y^2)^{1/2},$$

$$X = r^2 + p^2 - q^2, \quad Y = 2pq, \quad Q(\lambda) = \frac{16\beta\delta^2}{\delta^4 + 16\alpha^2\beta^2},$$

$$\alpha = \sqrt{1-\gamma^2\lambda^2}, \quad \beta = \sqrt{\lambda^2-1}, \quad \delta = 2-\lambda^2,$$

$\vartheta = c_R/c_S$, c_R is the Rayleigh wave velocity; here p_α is the sum of two waves of identical shape emitted by the true and imaginary sources, where the average stresses in the two waves have opposite signs. For example, if the wave from the true source is a compression wave, then the wave from the imaginary source is a tension wave. As $\gamma \to 0$, we find that p_α passes into the solution of an acoustic problem on an explosion in a liquid half-space; accordingly, p_α may be called the acoustic (quasiacoustic) term.

The term p_0 varies most slowly with respect to time; as $t \to \infty$, we find that $p_0 \to$ const (r, z, h), and therefore we may call it the quasistatic term. The term p_1 is due to the residues at the poles (the integrand in (4.2)), which are the roots of the Rayleigh equation; p_1 may be called the Rayleigh term, which describes the average stress in the Rayleigh surface wave.

The expression $\frac{3-4\gamma^2}{3}\frac{f''(t-\rho_2)}{\rho_2} + p_2$ is the average stress in the longitudinal body wave reflected from the free surface. The first term of this expression is the wave reflected from the free surface of what we might consider a liquid half-space, and p_2 is the additive term due to the difference between an elastic solid and a liquid.

The amplitude values of the individual terms in (4.3) depend on the coordinates of the point of observation and the depth of the source. Fixing the depth h of the source and letting the coordinate r tend to infinity, we can convince ourselves that the amplitudes of the

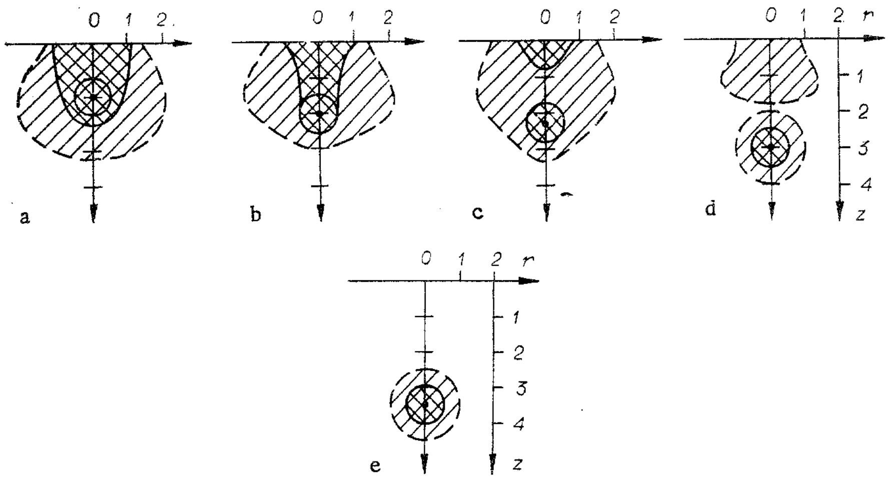

Fig. 2

individual terms in (4.3) have the following orders of magnitude:

$$p_a = O(1/r^2),\ p_0 = O(1/r^2),\ p_1 = O(1/\sqrt{r}),\ p_2 = O(1/r^2).$$

Consequently at long distances from the epicenter near the free surface the Rayleigh term p_1 will predominate. Conversely, at short distances from the epicenter and small depths of the explosion the biggest contribution to the average stress will be made by the quasistatic term p_0, which varies with distance according to a $1/\rho_2^3$ law.

We can expect that at distances $\rho_1 \sim \rho_2 \sim 1$ the principal part of p (where $\gamma \neq 0$) will be the following:

$$p \approx p_0 = \frac{3-4\gamma^2}{3}\,\frac{4\gamma^2}{1-\gamma^2}\left(-1+\frac{3z_2^2}{\rho_2^2}\right)\left[\frac{f'(t-\rho_2)}{\rho_2^2}+\frac{f(t-\rho_2)}{\rho_2^3}\right]. \tag{4.4}$$

If we try to take account of all possible values of the parameter γ $(0 \leqslant \gamma \leqslant 1/\sqrt{2})$, we find that the principal part of the average stress at short distances from the point of the explosion can be represented in the form

$$p \approx p_a + p_0 = \frac{3-4\gamma^2}{3}\left\{-\frac{f''(t-\rho_1)}{\rho_1}+\frac{f''(t-\rho_2)}{\rho_2}-\frac{4\gamma^2}{1-\gamma^2}\left(-1+3\frac{z_2^2}{\rho_2^2}\right)\left[\frac{f'(t-\rho_2)}{\rho_2^2}+\frac{f(t-\rho_2)}{\rho_2^3}\right]\right\}. \tag{4.5}$$

The linear dimensions of the region of possible disintegration of the medium when an explosion takes place (on the scale we have selected) have values of the order of unity (in the case of a camouflet explosion the radius of the zone of radial cracks is a quantity approximately equal to $c_p t_0$, while for an explosion in a half-space it is somewhat larger).

Consequently, in the region of possible spalling disintegration of a rock the average stress can be represented by (4.4) or (4.5).

Calculations made in accordance with the formulas (4.3) confirmed the above assertions with regard to the behavior of p (Fig. 1, where the amplitudes of the individual terms of (4.3) are shown for h = 2, B = 0.240, and γ = 0.6; curve 1 — p_m, 2 — p_{0m}, 3 — p_{1m}, 4 — p_{2m}).

5. Let us find the boundaries of the region within which it is possible to have disintegration of the medium. In this region the following condition must be satisfied:

$$p(r, z, t) \geqslant p_* \quad \left(p_* = \frac{1}{3}\sigma^+\right). \tag{5.1}$$

The equal sign in (5.1) corresponds to the boundary of the region of possible disintegration.

To estimate the "strength" of the medium, p_*, we use data on the dimensions of the disintegration zones in camouflet explosions. In a camouflet explosion the average stress, in

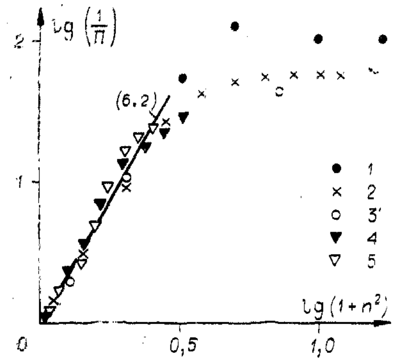

Fig. 3

accordance with (1.1), (4.1), is described by the formula

$$p = p(R, t) = \frac{3-4\gamma^2}{3}\frac{\partial^2\varphi_0}{\partial t^2} = -\frac{3-4\gamma^2}{3}\frac{f''(t-R)}{R},$$

where R is the distance from the center of the explosion; f has the form $R = \tilde{R}/c_p t_0$.

The zone of radial cracks in a spherically symmetric explosion in rock begins at distances of $\tilde{R}_1 \approx 2r_c$ and ends at distances of $\tilde{R}_2 \approx 4r_c$ from the center of the explosion (where r_c is the radius of the camouflet cavity) [1, 13]. On the chosen scale of length these distances will be $R_1 \approx 0.5$ and $R_2 \approx 1.0$, respectively.

Since crack formation in rocks in the zone of radial cracks takes place as a result of tension, a knowledge of the limits R_1 and R_2 of this zone enables us to estimate the strength of the rock under tension when an explosion takes place.

For the value of the disintegrating average stress we obtain the following limits:

$$p_1^* \equiv -\frac{3-4\gamma^2}{3}\frac{f_m''}{R_1} \geqslant p_* \geqslant -\frac{3-4\gamma^2}{3}\frac{f_m''}{R_2} \equiv p_2^*, \qquad (5.2)$$

where f_m'' is the largest value of the second derivative of the source function (1.2) (in the tension phase in the wave emitted by the explosion). If we take for granite a value of $\gamma = 0.6$ (and, in accordance with [5], $B = 0.240$), we can find the value $f_m'' \approx 0.38$, which, taking account of (5.2), yields

$$p_1^* \approx 0.4 \quad (R_1 = 0.5), \quad p_2^* \approx 0.2 \quad (R_2 = 1.0). \qquad (5.3)$$

If in (5.1) we use the value of p from (4.3) and the limits found in (5.3) for the "strength" of the granite, we can calculate the boundaries of the possible spalling disintegration in the case of an explosion in granite.

The results of such calculations are shown in Fig. 2. The solid curves indicate the boundaries of possible disintegration when $p_* = 0.4$, and the dashed curves indicate the boundaries for $p_* = 0.2$ for different values of the source depth h, a) h = 1.5 b) 2.0, c) 2.25, d) 3.0, e) 3.5.

The common characteristic feature of the zones of possible spalling disintegration is their localization about the axis of symmetry and their elongation along this axis. The tensile stresses resulting from the presence of a free surface disintegrate the medium preferentially along the line of least resistance.

This fact makes understandable the phenomenon of the formation a cave-in column of rock and a collapse funnel when an explosion takes place. For appropriate explosion depths, the rock situated above the camouflet cavity and disintegrated by the direct and reflected waves, under the action of gravity, gradually collapses into the camouflet cavity. The cavity may be said to be uplifted to the free surface, becoming a collapse funnel.

In particular, it can be said that if the zones of spalling disintegration and camouflet disintegration do not overlap, the cave-in column will not emerge to the free surface and a collapse funnel cannot be formed.

6. We shall make use of the foregoing to obtain a formula relating the radius of the disintegration funnel at the free surface to the weight of the charge and depth of its placement.

The edge of the disintegration funnel at the free surface of the medium is characterized by the fact that the amplitude p_m of the average stress on it in accordance with the disintegration criterion (5.1) must be equal to the "strength" of the medium:

$$p_m(r, 0, h) = p_*. \tag{6.1}$$

In order to obtain the explicit expression in (6.1), we carried out stress calculations according to the formulas (4.3). The results are shown in the coordinates log (Π^{-1}), log $(1 + n^2)$ in Fig. 3, where $\Pi = p_m(r, 0, h)/p_m(0, 0, h)$, $n = r/h$, $p_m(0, 0, h)$ is the amplitude at the epicenter ($B = 0.240$, $\gamma = 0.6$). It can be seen from Fig. 3 that the distributions of the function Π along the free surface for different values of h are approximately the same [at point 1) h = 1; 2) h = 1.5; 3) h = 2; 4) h = 3; 5) h = 4].

In the interval $0 < n \leqslant 1.28$ the results can be approximated very well by a straight line:

$$\lg (\Pi^{-1}) = a \lg (1 + n^2), \ a \approx 3.76. \tag{6.2}$$

When $n \geqslant 1.28$, the points corresponding to different values of h lie on different curves, and the approximation of the function log (Π^{-1}) in this interval in the form of a single unified curve is incorrect. These facts indicate that as n changes, there is a change in the behavior of the function Π.

The calculations for the individual terms in (4.3) show that when $0 < n < 1.28$, the main contribution to the value of the amplitude of the average tension is made by the quasistatic term p_{0m}, and when $n \geqslant 1.28$, it is made by the Rayleigh term p_{1m}. In the neighborhood of the value $n \approx 1.28$ the regime of motion of the free surface is altered. For small values of n the largest average tension is contributed by the body waves, and for large values ($n > 1.28$) by the Rayleigh surface wave (see Fig. 1). The amplitude of the latter wave depends not only on n but also on h, i.e., the Rayleigh waves for different source depths are not similar to each other.

The results of the calculation of the amplitude at the epicenter showed that the value $p_m(0, 0, h)$ can be satisfactorily approximated by a power function:

$$p_m(0, 0, h) = A/h^m, \tag{6.3}$$

where $m \approx 2.80$; $A \approx 4.56$ $(0.5 < h < 5.0)$.

From (6.1)-(6.3) we find

$$A/h^m (1 + n^2)^a = p_*. \tag{6.4}$$

Returning to the dimensional variables $H = c_p t_0 h$, $R = c_p t_0 r$, we rewrite the condition (6.4) in the form

$$c_p t_0 = (p_*/A)^{1/m} (1 + n^2)^{a/m} H. \tag{6.5}$$

The characteristic length $c_p t_0$ is proportional to the radius of the camouflet cavity: $c_p t_0 \sim r_c$. The radius of the cavity can be estimated by means of the formula [14]

$$r_c = \text{const} (W^{1/3}/H^{1/4}), \text{where } W \text{ is the weight of the charge.} \tag{6.6}$$

Making use of (6.6), we find from (6.5) that

$$W = \text{const} \left(p_*/\varkappa\rho_0 c_p^2\right) (1 + n^2)^{3a/m} H^{15/4} \approx \text{const} \left(p_*/\varkappa\rho_0 c_p^2\right) (1 + n^2)^4 H^{15/4}. \tag{6.7}$$

Formula (6.7) yields the desired connection between the weight of the charge, the depth of its placement, the strength of the medium, and the radius of the disintegration funnel for values of $0 \leqslant n \leqslant 1.28$. For larger values of n this connection is much more complicated.

In ejection explosions the radius of the ejection funnel practically coincides with the radius of the disintegration funnel, and therefore formula (6.7) can be used for calculating the ejection funnel. The ratio $n = R/H$, where R is the radius of the ejection funnel, is called the ejection factor [1]. If in formula (6.7) we set $H = R/n$, we obtain

$$W = \text{const} \left(p_*/\varkappa\rho_0 c_p^2\right) \left((1 + n^2)^4/n^{-15/4}\right) R^{15/4}. \tag{6.8}$$

From (6.8) we conclude that there exists an optimal depth of placement, $H = H_*$. When the explosion takes place at this depth, the weight of charge required to produce a funnel of a given radius is a minimum. Differentiating (6.8) with respect to n, setting the derivative equal to zero, and solving the resulting equation for n, we find the value of the optimum ejection factor

$$n_* = \left(\frac{15}{17}\right)^{1/2} \approx 0.94.$$

Experiments with large explosions show that the optimal value of the ejection factor depends weakly on the type of rock involved and is close to unity [13, 15]:

$$n_* \approx 0.8 - 1.2.$$

If we disregard the effect of the depth of placement upon the length of the elastic wave emitted by the explosion (which is permissible for small explosions), then the characteristic length $c_p t_0$ is proportional to $W^{1/3}$. Formula (6.5) then yields

$$W = \mathrm{const}\,(p_*/\varkappa\rho_0 c_p^2)\,(1 + n^2)^4 H^3,$$

i.e., the weight of charge required to produce an ejection funnel is proportional to the cube of the line of least resistance.

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